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TARGET TRACKING WITH RANDOMLY INTERRUPTED RANGE DATA WITH APPLI--ETC(U)

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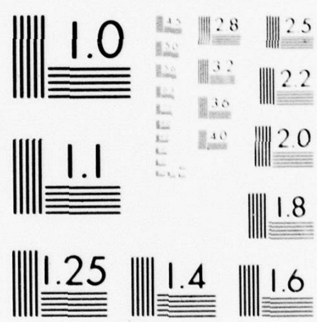
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The effect of randomly missed range data on target tracking performance has been investigated. The specific problem addressed concerns SEAFIRE. Most of the development and results, however, are general enough to be extended to other purposes.

Randomness of the missing range data was modeled as a first order Markov process while Singer's correlated acceleration model was used to represent

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the target. Both of these models provide a quite general description of the stochastic processes being analyzed. The effect of uniformly (non-random) missed range data was also explored in order to provide a reference which is somewhat easier to analyze and understand. For the sake of completeness, angle tracking performance was briefly examined for the case of uninterrupted angle measurements.

Results presented in this technical report are designed to aid in determining performance to be expected from a target tracking algorithm subjected to randomly interrupted range data and to aid in specifying sensor accuracy requirements necessary to obtain the desired tracking accuracy.

FOREWORD

This study was performed in the Weapons Control Division of the Combat Systems Department. The main objective was to investigate the effect of randomly missed range data on filter/predictor performance for a SEAFIRE target tracking algorithm. The data resulting from this investigation is designed to aid in determining system (sensor, etc.) requirements for various degrees of target tracking performance.

The work presented in this report was supported by the SEAFIRE Project Office. This report has been reviewed by Edward L. Price and Robert A. Lindeman of the Systems Engineering Branch.

Released by:

G. O. MILLER, Head

Combat Systems Department

CONTENTS

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I. INTRODUCTION

The primary purpose of a tracking filter in any fire control system is to process sensor information to obtain target state estimates (position, velocity, etc.) suitable for accurately predicting future target position. Designing a tracking filter/predictor algorithm is fairly straight-forward once a target model and sensor error model have been established. The above is true even for the case of randomly interrupted range data. Generally, the tracking filter is designed to simply "coast" during the data interruption. Error analysis for such a system, however, is quite difficult and is the major topic addressed in this technical report.

For the case of uninterrupted data (assuming the usual linear target and measurement model and linear estimation algorithm), filter/predictor error covariance (a matrix describing the random nature of filter/predictor estimates) is deterministic and can be easily calculated. For the case of randomly interrupted data, however, the error covariance is not determinisitc but a random process. It is precisely this property that makes error analysis of the interrupted data system difficult.

The analysis presented in this report computes the first moment (expected value) of the filter/predictor error covariance using an exact and an approximate technique. The approximate technique is necessary because of the tremendous computational burden associated with the exact technique.

Most of the analysis presented is directed toward range filtering with randomly interrupted range data. A brief analysis, however, is presented for angle filtering with uninterrupted angle data.

II. TARGET MODEL

The target model assumed in this study is a correlated acceleration model developed for target tracking analysis by Singer in 1970. This model was chosen because of its simplicity, flexibility and previous successful filter design applications such as the GIP filter (reference 2). An uncoupled spherical coordinate system was chosen also because of simplicity and previous successful application (reference 3). As an example of this technique, the range direction modeling assumptions are illustrated by the following equation.

$$\dddot{r} = \frac{-\ddot{r}}{\tau_m} + \omega(t) \tag{1}$$

where r is target range, $\omega(t)$ is a white Gaussian process, τ_m is a serial "time constant" and superscript "$\dot{}$" denotes time derivative.

Statistical properties of the above process are conveniently described by the autocorrelation function

$$R_{\ddot{r}}(t) \overset{\Delta}{=} E[\ddot{r}(t+t_o)\ddot{r}(t_o)] = \sigma_m^2\, \varepsilon^{\frac{-|t|}{\tau_m}} \tag{2}$$

where $E(\cdot)$ denotes statistical expected value operation. The parameter σ_m of equation (2) specifies the target "maneuver level" (references 1,2).

Modeling for the elevation and bearing directions is identical to range direction modeling except for slight differences which are described in reference 1.

III. MODELING THE MISSED DATA PROCESS

A first order Markov process was chosen to model the randomness of the missing data process. Such a process is reasonably versatile in that it allows the probability of receiving a measurement and correlation time to be chosen independently. Also, by using the Chapman-Kolmogoroff equation as described in reference 4, propagation of measurement probabilities can be conveniently described by

$$\begin{bmatrix} p(m_{k+1}) \\ p(\bar{m}_{k+1}) \end{bmatrix} = \Upsilon \begin{bmatrix} p(m_k) \\ p(\bar{m}_k) \end{bmatrix} \tag{4}$$

where $p(m_i)$ denotes probability of receiving a measurement at time t_i and $p(\bar{m}_i)$ denotes the probability of not receiving a measurement at time t_i.

The 2x2 transition matrix is given by

$$\Upsilon = \begin{bmatrix} p(m_{k+1}|m_k) & p(m_{k+1}|\bar{m}_k) \\ p(\bar{m}_{k+1}|m_k) & p(\bar{m}_{k+1}|\bar{m}_k) \end{bmatrix} \tag{5}$$

where the bar "|" indicates a conditional probability, i.e., $p(\bar{m}_{k+1}|m_k)$ denotes probability of not receiving a measurement at time t_{k+1} given a measurement was received at time t_k. Appendix A describes how the elements of Υ are chosen to match the properties of a physical system.

IV. TRACKING (FILTER/PREDICTOR) ALGORITHM

This study assumes a Kalman tracking filter designed about the uncoupled spherical target modeling scheme discussed earlier. Figure 1 illustrates the structure of such a filtering technique via block diagram. The indicated coupling between filters is included to allow constant maneuverability modeling as discussed in reference 1. The coupling is quite "loose", however, as demonstrated by Quigley (reference 3) where a successful tracking algorithm was designed by completely ignoring the coupling.

Also, it is assumed the prediction function is performed via the simple state transition method. This technique (in spherical coordinates) was demonstrated to be substantially superior (for point defense applications (reference 3)) to similar techniques in Cartesian coordinates.

The analysis presented in this note pertains specifically to the filter/predictor assumptions described above. The results, however, are quite indicative of performance to be expected from other well designed tracking schemes addressing the same problem.

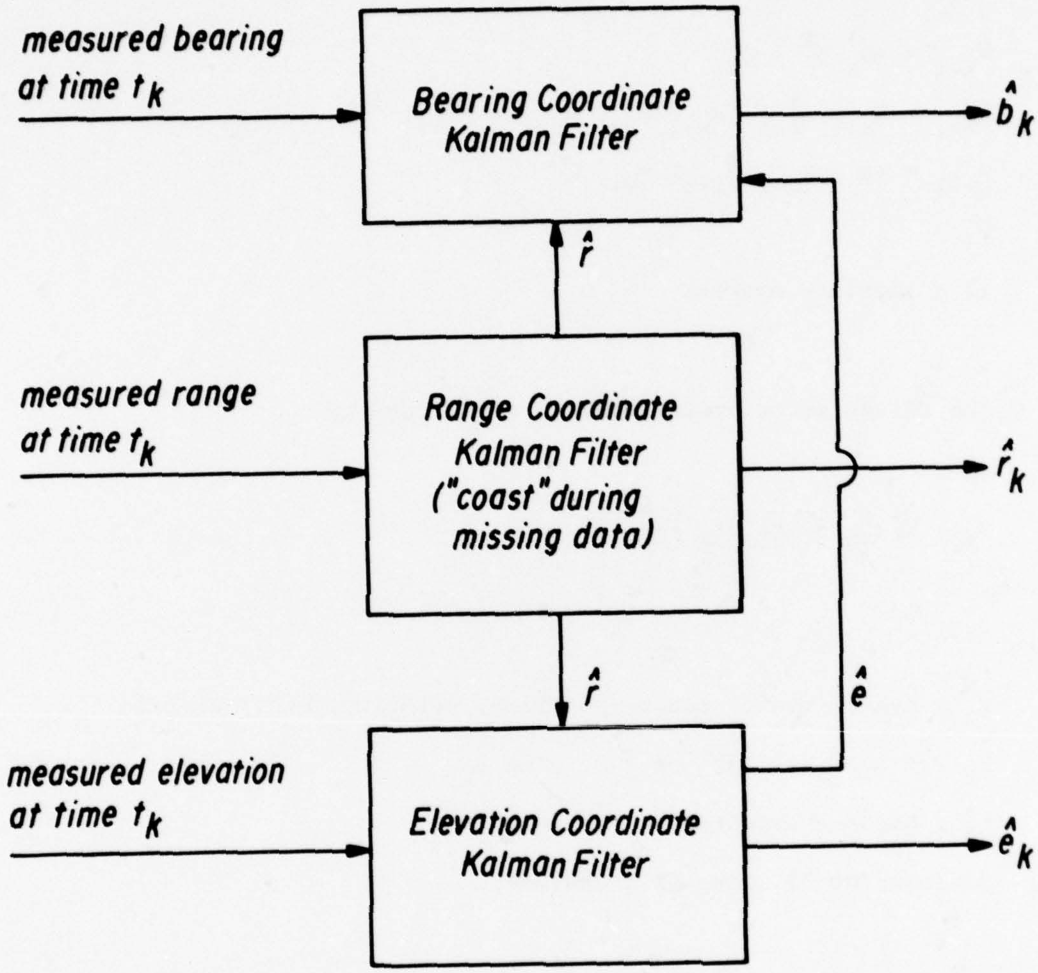

Figure 1. Kalman Tracking Algorithm Using Uncoupled Spherical Model (Block Diagram)

V. EXACT SOLUTION FOR ERROR COVARIANCE WITH RANDOMLY MISSED DATA

The error covariance matrix of a Kalman filter algorithm is propagated (in time) by the following equations:

$$
\begin{aligned}
M_{k+1} &= \phi P_k \phi^T + Q_k \\
K_{k+1} &= M_{k+1} H_{k+1}^T \left(H_{k+1} M_{k+1} H_{k+1}^T + R_{k+1}\right)^{-1} \\
P_{k+1} &= \left(I - K_{k+1} H_{k+1}\right) M_{k+1}
\end{aligned}
\tag{6}
$$

(I = identity matrix)

The filter error covariance, P, is defined by

$$P_k \triangleq E\left[(\hat{\underline{X}}_k - \underline{X}_k)(\hat{\underline{X}}_k - \underline{X}_k)^T\right]$$

where

$\underline{X}_k$ = system state vector (position, velocity, etc.) at time t_k.

$\hat{\underline{X}}_k$ = filter estimate of $\underline{X}$ at time t_k.

$E[\cdot]$ denotes expected value operation.

Superscript "T" denotes transpose.

The matrix quantities ϕ, R and Q are the system state transition matrix, measurement error covariance matrix and process disturbance covariance matrix respectively.

Definitions and physical significance of ϕ, R and Q are described in references 5 and 6 while Singer (reference 1) describes the exact formulation of these parameters for the target model being considered.

The matrix H of equations (6) is the observation matrix which indicates how measurements are related to the system state vector. As an example, consider that noisy measurements of range ($Z^{(r)}$) are made and the range coordinate state vector consists of range (r), range rate ($\dot{r}$) and range acceleration ($\ddot{r}$). The measurement equation becomes

$$
\begin{aligned}
Z^{(r)}_{k+1} &= H_{k+1} \underline{X}_{k+1} + V_{k+1} \\
&= [1 \mid 0 \mid 0] \begin{bmatrix} r \\ \dot{r} \\ \ddot{r} \end{bmatrix} + V_{k+1}
\end{aligned} \tag{7}
$$

where V_{k+1} represents the error in measured range at time t_{k+1}. Notice that if the measurement at time t_{k+1} is missed, H_{k+1} becomes all zero and equations (6) reduce to

$$P_{k+1} = \phi P_k \phi^T + Q_k. \tag{8}$$

For the case of uninterrupted data (or non-randomly interrupted data) the filter error covariance is a deterministic quantity fully described by equations (6). For the case of randomly interrupted data, however, the situation is considerably more complex since error covariance becomes a random process. An exact solution for the expected value of filter error covariance is easily formulated as the weighted average of all possible filter error covariances. Such a formulation is given by

$$E[P_{k+1}] = \sum_{\text{all cases}} P_{k+1}(\text{case}_i)\ \text{Prob}(\text{case}_i) \tag{9}$$

where $case_i$ denotes the i^{th} possible measurement sequence and $Prob(case_i)$ denotes the probability of $case_i$ occurring. To clarify the above, consider the simple example where P_o is known and the expected value of P_1 is desired. The following two measurement sequences (cases) are possible.

$Case_1$ measurement <u>does not</u> occur at time t_1.

$Case_2$ measurement <u>does</u> occur at time t_1.

From equations (6), $P_1(case_i)$ for i = 1,2 can be formulated to yield

$$P_1(case_1) = \phi P_o \phi^T + Q_o$$

$$P_1(case_2) = (I - K_1 H_1) M_1$$

where $M_1 = \phi P_o \phi^T + Q_o$

$$K_1 = M_1 H_1^T (H_1 M_1 H_1^T + R_1)^{-1}.$$

Assuming the probability of missing a measurement at time t_1 is q_1 (which implies the probability of receiving a measurement at time t_1 is $1 - q_1$) results in the following exact expression for the expected value of P_1.

$$E[P_1] = [(1-q_1)\ (I-K_1H_1)M_1] + q_1[\phi P_o \phi^T + Q_o\ ]$$

where K_1 and M_1 are defined earlier. From this simple example, it is easy to see the difficulty in implementing equation (9) for k fairly large. Specifically, the complexity of equation (9) increases exponentially with increasing time steps. For the general case,

$$E[P_{k+1}] = \sum_{i=1}^{i=2^{k+1}} P_{k+1}(case_i)\ Prob(case_i).$$

Prediction

The prediction error covariance equations are linear and therefore allow for a simple solution for the expected value. Specifically, the expected value of prediction error covariance is given by

$$E[M(t_p)] = \phi(t_p)E[P_k]\phi^T(t_p) + Q(t_p)$$

where

$$M(t_p) \triangleq E[(\underline{\hat{X}}(t_k+t_p) - \underline{X}(t_k+t_p))(\underline{\hat{X}}(t_k+t_p) - \underline{X}(t_k+t_p))^T]$$

t_p = prediction time

t_k = time of last measurement

Restrictions

In practice, it has been found that equation (9) has practical use for k ≈ 10 or less. It is precisely this computational restriction that motivated development of the recursive approximate solution presented in the next section.

VI. RECURSIVE APPROXIMATE SOLUTION FOR ERROR COVARIANCE WITH RANDOMLY MISSED DATA

The covariance propagation equations (equations (6)) can be combined to yield one nonlinear equation for the case of measurement occurring at time t_{k+1}. This equation is shown below in an explicit and an abbreviated form.

$$P_{k+1} = [I - ((\phi P_k \phi^T + Q_k) H_{k+1}^T (H_{k+1} (\phi P_k \phi^T + Q_k) H_{k+1}^T + R_{k+1})^{-1}) H_{k+1}] \cdot [\phi P_k \phi^T + Q_k] \qquad (11)$$

$$P_{k+1} = f_{NL}(P_k) \qquad (12)$$

where $f_{NL}(\cdot)$ represents the nonlinear function described in equation (11). From equation (8) (case of no measurement at time t_{k+1}) we have the linear equation

$$P_{k+1} = \phi P_k \phi^T + Q_k \qquad (13)$$

$$P_{k+1} = f_L(P_k) \qquad (14)$$

where $f_L(\cdot)$ represents the linear function described in equation (13). Using the notation just defined, the expected value of P_{k+1} can be expressed by

$$E[P_{k+1}] = E[f_{NL}(P_k)] \ \text{prob}(m_{k+1}) + E[f_L(P_k)] \ \text{prob}(\bar{m}_{k+1}) \qquad (15)$$

where prob(m_{k+1}) denotes the probability of receiving a measurement at time t_{k+1} and prob($\overline{m}_{k+1}$) denotes the probability of not receiving a measurement at time t_{k+1}. To generate the recursive approximate solution, the following approximation is needed.

$$E[f_{NL}(P_k)] \approx f_{NL}(E(P_k))$$

This approximation allows equation (15) to be rewritten yielding the very simple recursive solution

$$E[P_{k+1}] \approx f_{NL}(E(P_k))\text{prob}(m_{k+1}) + f_L(E(P_k)) \cdot \text{prob}(\overline{m}_{k+1}) \tag{16}$$

Prediction

Expected value of the prediction error covariance is obtained exactly as for the exact solution of the previous section.

Restrictions

This approximate solution effectively ignores 2nd and higher order statistical moments of the matrix P. Appendix B presents experiments conducted to determine accuracy of the recursive approximate solution (equation (16)). From these experiments, it was concluded the approximate solution is quite accurate enough for the intended purposes.

VII. RESULTS

This section primarily seeks to describe the effect of missing range data on tracking filter/predictor performance. A very brief analysis of angle tracking is also presented. The cases of randomly missed and uniformly missed range data are explored. Experiments were conducted involving surface and air target models. Both models were Singer (reference 1) correlated acceleration models with the following parameters:

Air $\sigma_m = 0.5 \text{ yds/sec}^2$
$\tau_m = 20 \text{ sec}$

Surface $\sigma_m = 0.1 \text{ yds/sec}^2$
$\tau_m = 20 \text{ sec.}$

These parameters were chosen based on experience gained during the Gunnery Improvement Program (GIP) (reference 2). For both models, the uninterrupted data rate was assumed 16 Hz. The remainder of this section presents graphical summaries of the experiments conducted.

Randomly Interrupted Range Data

Results of these experiments were obtained using steady state solutions obtained via the recursive approximate technique described in the previous section (equation (16)) and in Appendix B. The results are presented by standard deviation prediction error contours. These contours present the data in a very compact form and therefore must be interpreted very carefully. Each contour represents a locus of necessary conditions for a particular performance criterion (standard deviation prediction

error in position). A contour may be interpreted as a plot of required probability of receiving a measurement as a function of sensor error standard deviation. Also, a contour may be interpreted as a plot of required standard deviation sensor error as a function of probability of receiving a measurement. Specific procedures for interpreting these contours are best illustrated by example.

Example 1: Refer to Figure 2 which addresses a prediction time of 25 sec for surface targets. Assume the sensor of interest provides range data with errors of 27.5 yds standard deviation. Locating 27.5 yds on the horizontal axis and projecting vertically results in intersecting the 40 yd contour at prob(measurement) = 0.7. This result indicates that the given sensor must provide probability of measurement $\geq$ 0.7 to achieve an expected standard deviation prediction error of 40 yds. Also, notice that the 30 yd contour is not intersected for the sensor of interest. This result indicates that such performance is not possible for the given sensor even if all measurements are received.

Example 2: Refer to Figure 3 which addresses prediction time of 10 sec for air targets. Assume the sensor of interest provides a probability of measurement = 0.9. Locating 0.9 on the vertical axis and projecting horizontally results in intersecting the 20, 30 and 40 yd contours at 3.5, 17.5 and 42 yds standard deviation sensor error. These results indicate that achieving standard deviation prediction errors of 20, 30 and 40 yds requires standard deviation sensor errors at least as good as 3.5, 17.5 and 42 yds respectively.

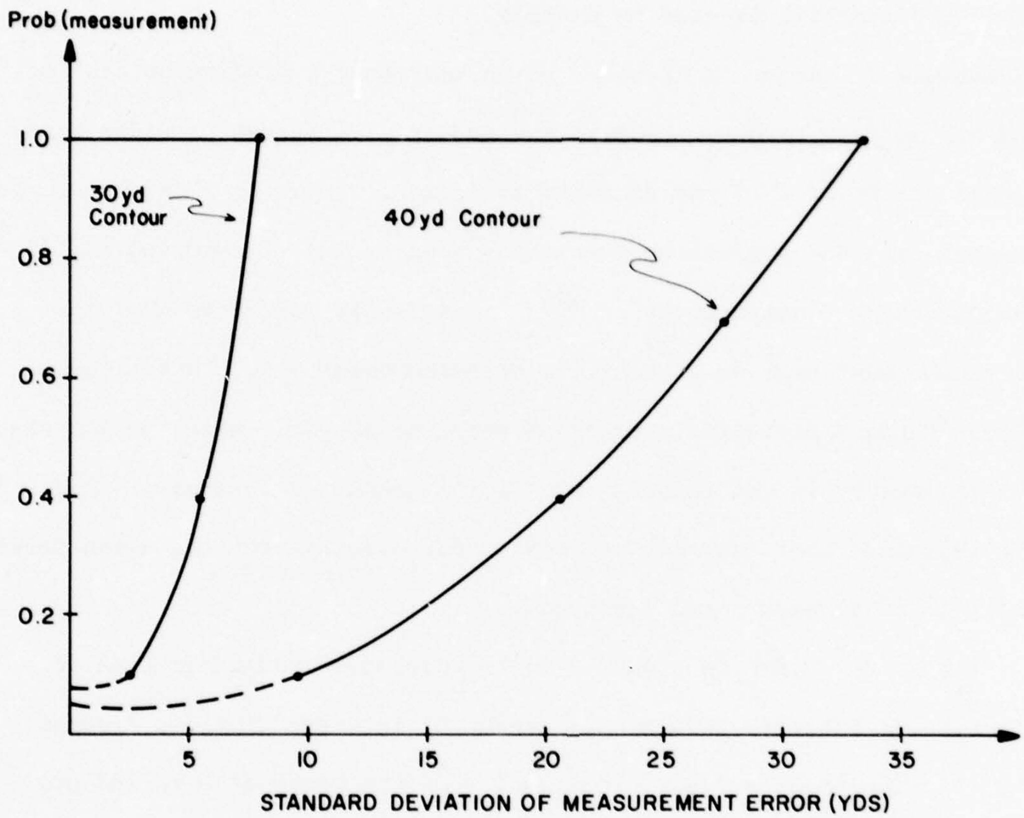

Figure 2. Standard Deviation Prediction Error Contours for Surface Target (25 sec prediction time)

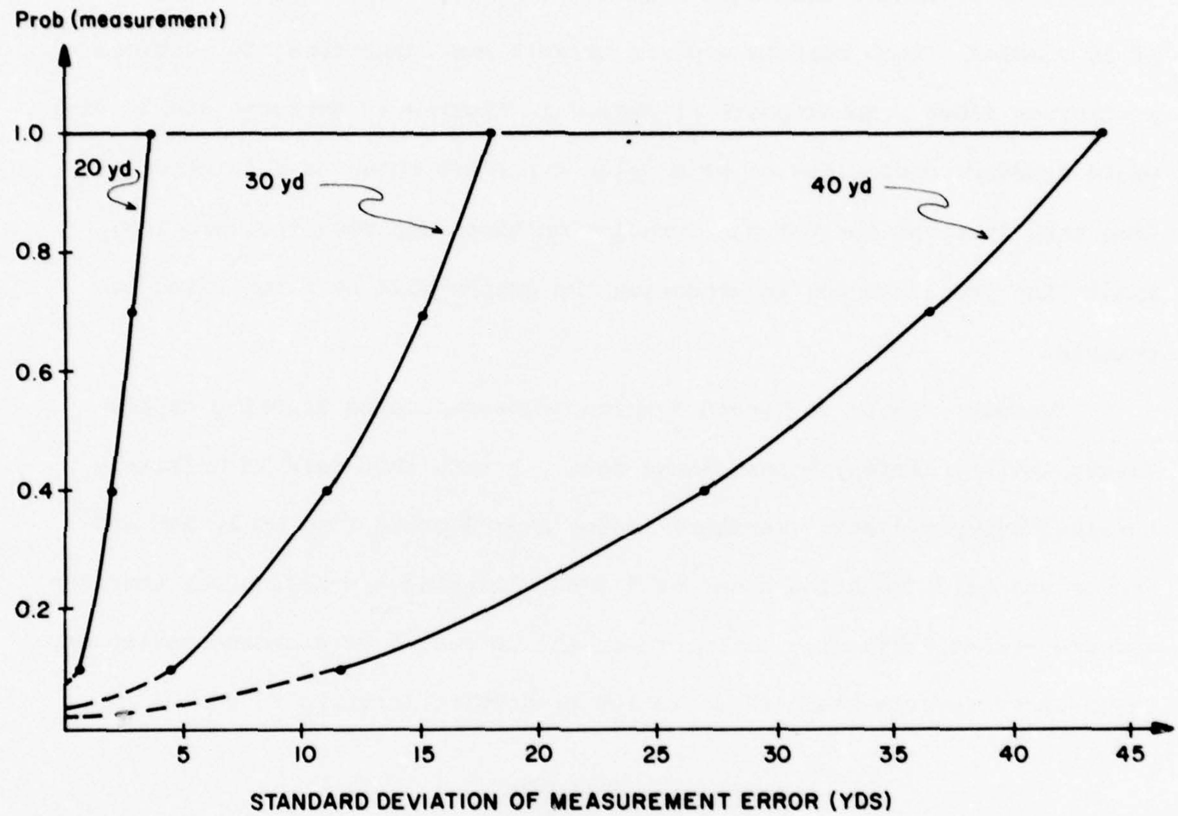

Figure 3. Standard Deviation Prediction Error Contours for Air Target (10 sec prediction time)

Uniformly Interrupted Range Data

Results of these experiments were obtained using steady state solutions obtained from the standard Kalman filter covariance equations (equations (6)). The time step between samples was varied from 1/16 sec to 1 sec to represent uniformly missing no data to uniformly missing 15 of 16 samples. Both surface and air targets were considered for various prediction times. Results are displayed in Figures 4 (surface) and 5 (air) where standard deviations of prediction error are shown as a function of time step interval for various prediction times and sensor accuracies. Again, the procedure for interpreting the graphs will be illustrated by example.

Example: Refer to Figure 5 which addresses range tracking of air targets with uniformly missed range data. Assume that data is uniformly received every 3/4 sec. Further, assume a prediction time of 10 sec and sensor standard deviation error of 5 yds. Locating 3/4 sec on the abscissa and projecting vertically to intersect the 10 sec, 5 yd standard deviation error curve reveals standard deviation prediction error is 32 yds.

Sensitivity Analysis

A brief sensitivity analysis was performed to determine the effect of τ_m and σ_m of Singer's target model (reference 1) on steady state prediction error variance. The main objective of this analysis was to examine sensitivity with respect to τ_m, since this parameter is the most difficult to specify. For convenience, the uniform missed data solution was used. Results are presented in Figures 6 through 9 where performance graphs, such as just discussed, are presented for various values of τ_m and σ_m. From these results it was concluded that sensitivity to τ_m is relatively mild.

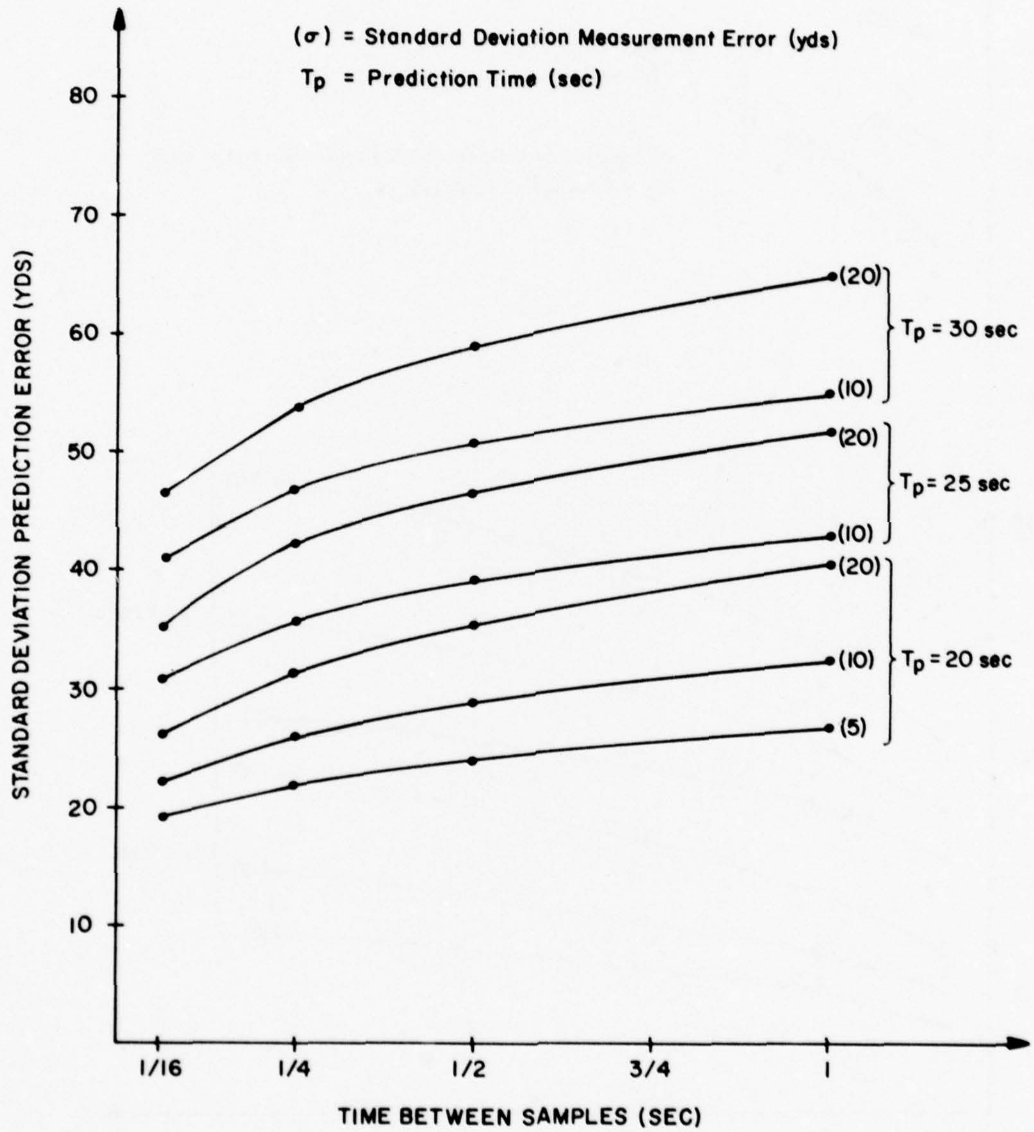

Figure 4. Uniformly Missed Range Data (Surface Target)

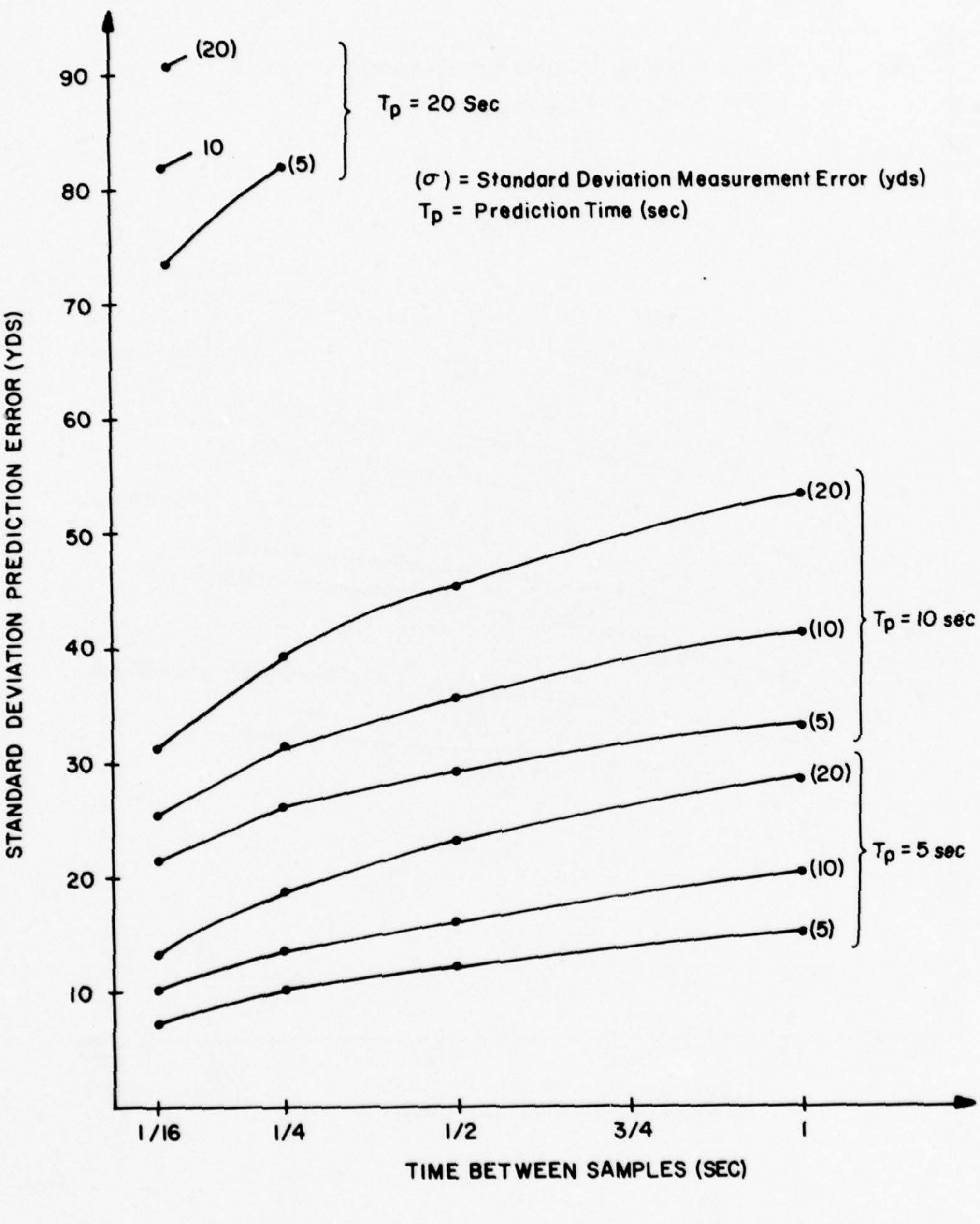

Figure 5. Uniformly Missed Range Data (air target)

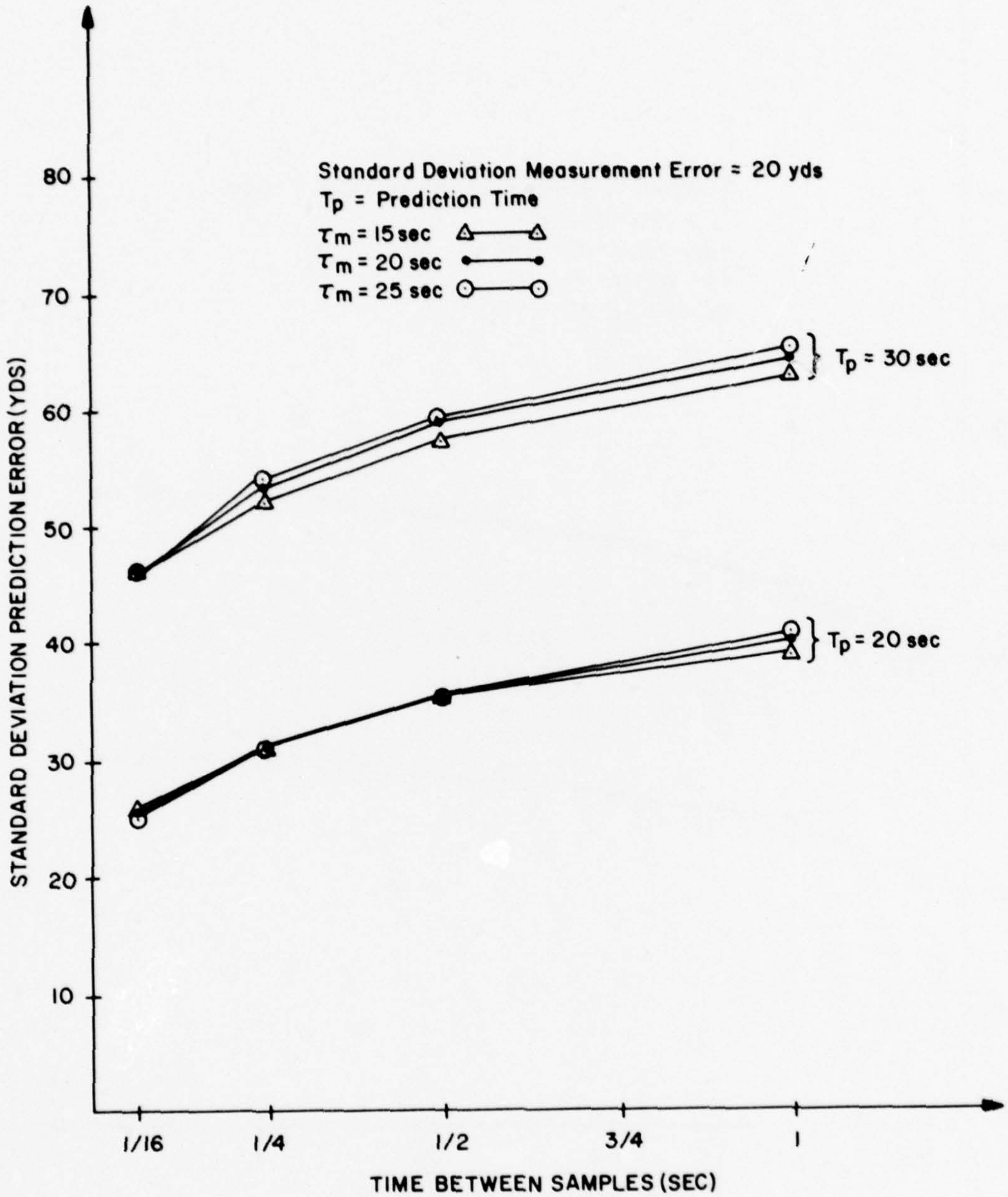

Figure 6. Prediction Error Covariance Sensitivity with Respect to τ_m (surface target)

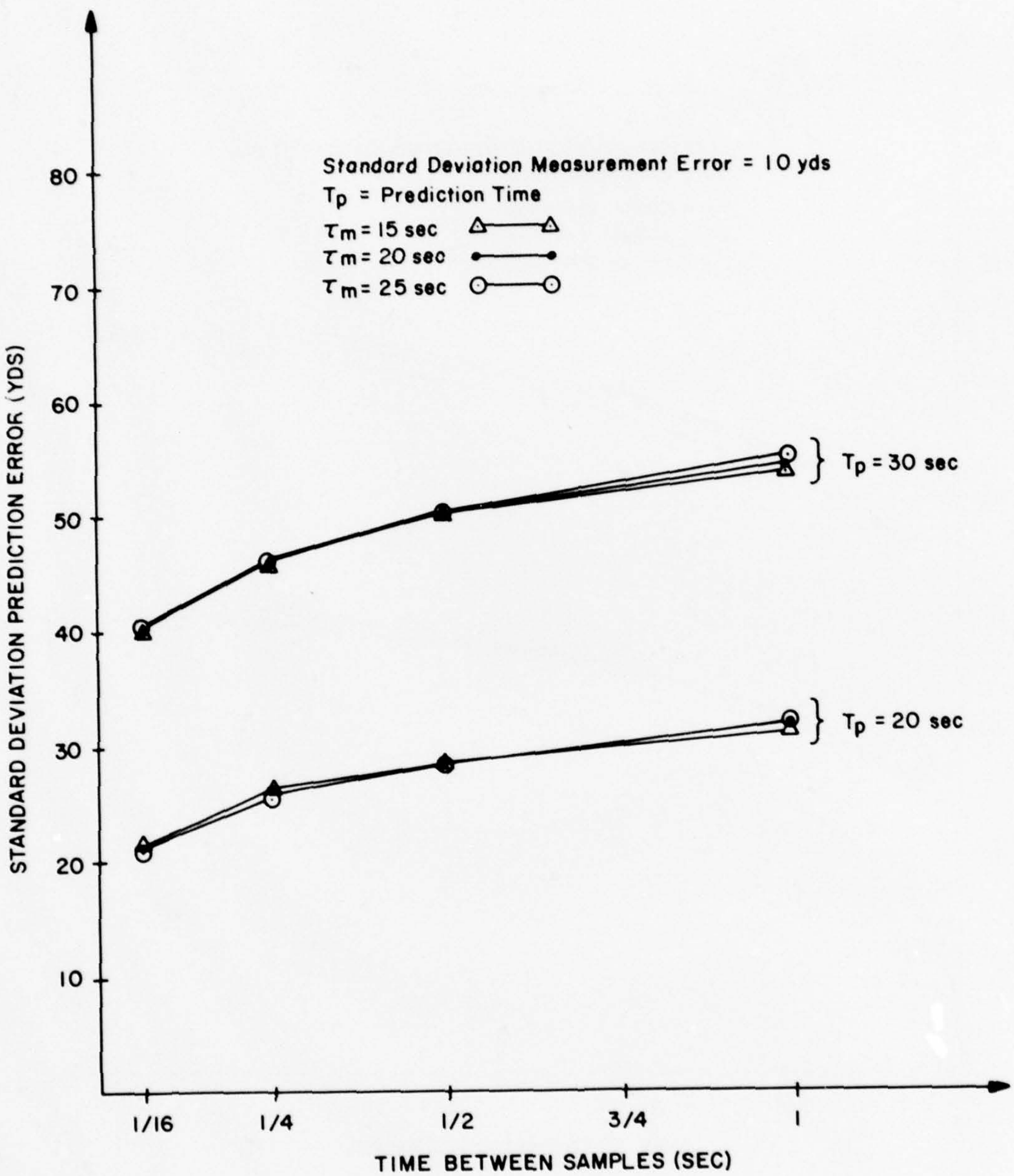

Figure 7. Prediction Error Covariance Sensitivity with Respect to τ_m (surface target)

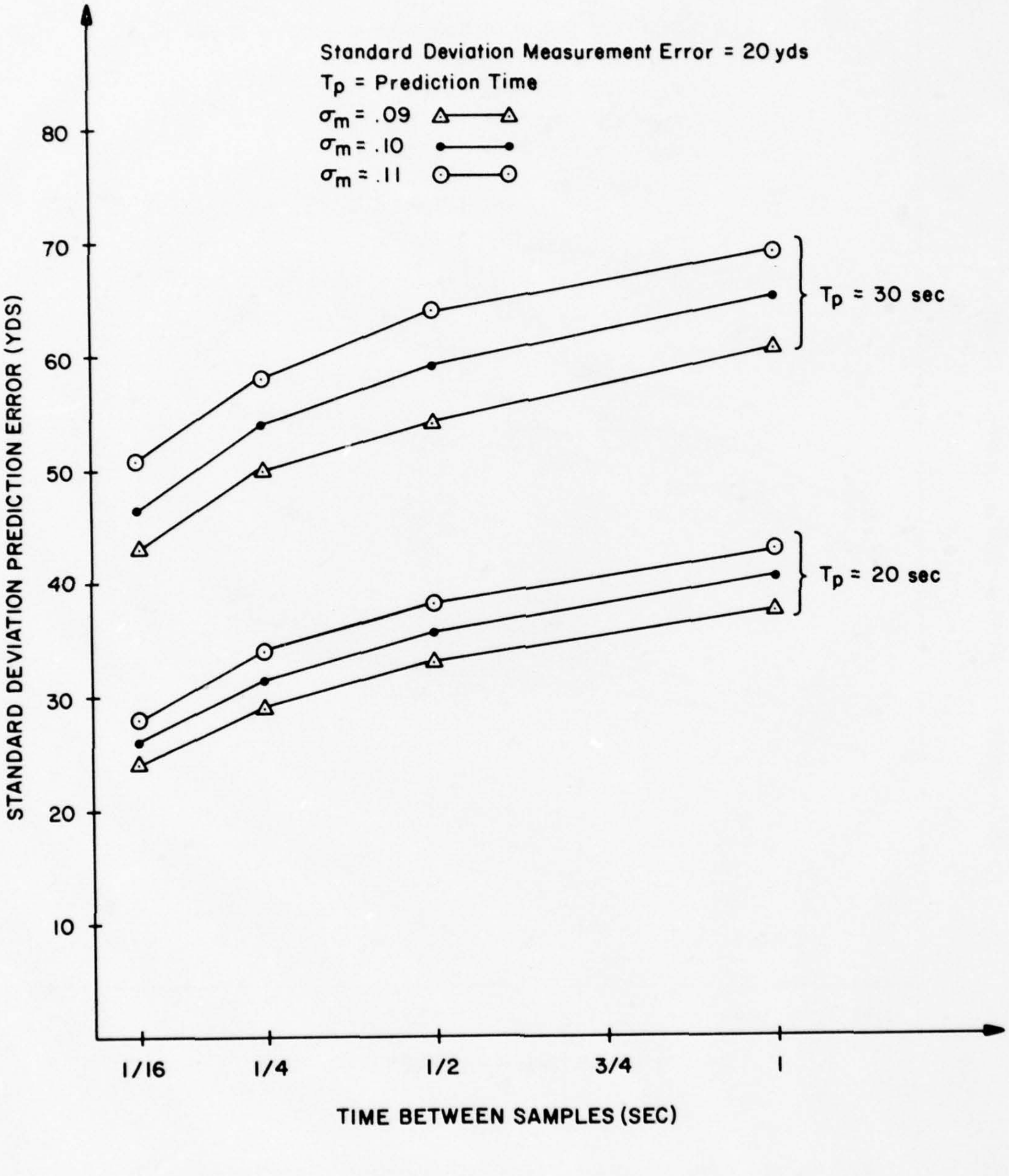

Figure 8. Prediction Error Covariance Sensitivity with Respect to σ_m (surface target)

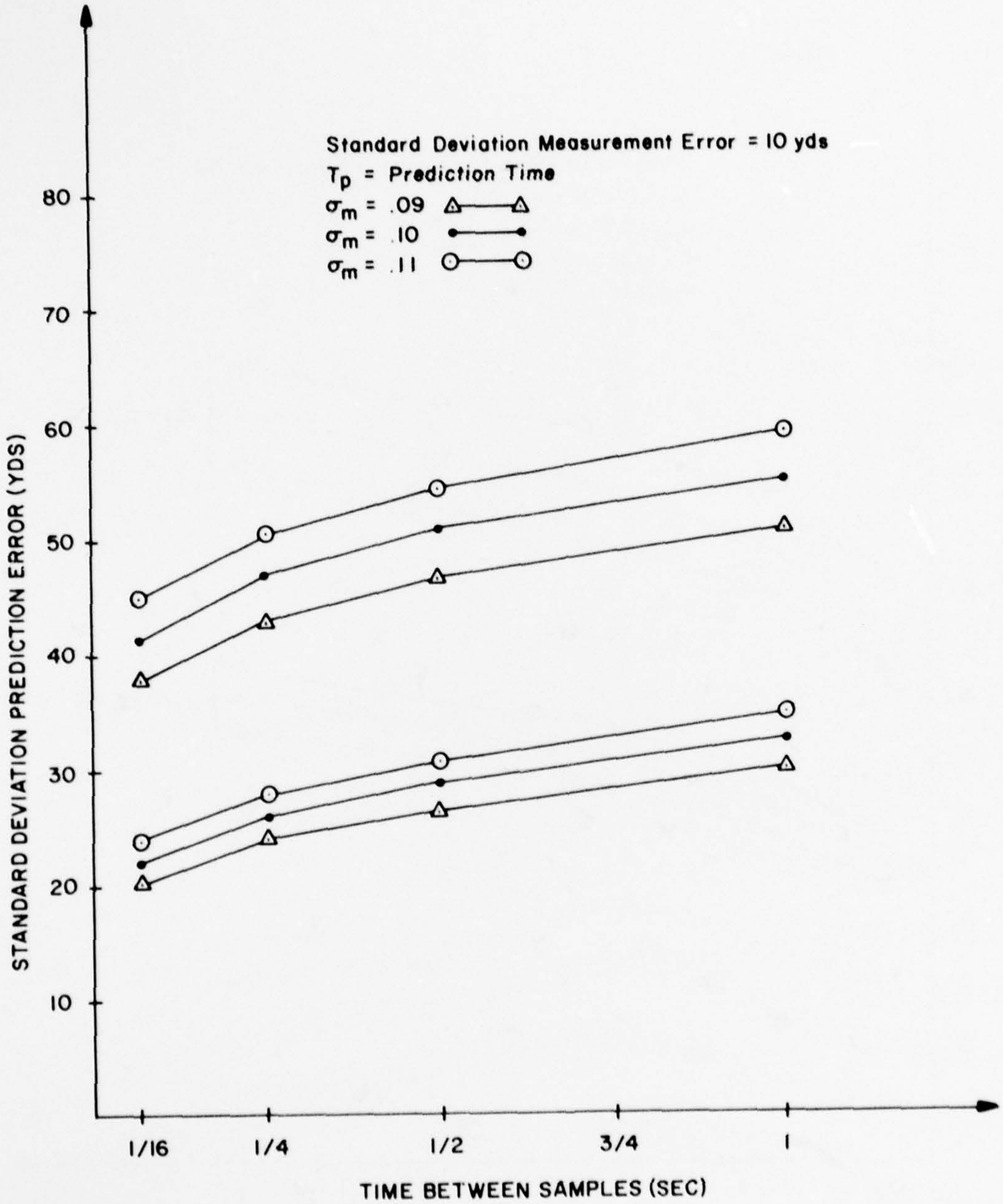

Figure 9. Prediction Error Covariance Sensitivity with Respect to σ_m (surface target)

Bearing Direction Analysis (uninterrupted data)

For the sake of completeness, a brief analysis was conducted to determine bearing direction prediction accuracy. The analysis was performed for surface mode only since an air target analysis would require assuming a specific target trajectory. Results are shown in Figure 10 where standard deviation of prediction error is plotted against prediction time for various sensor statistics. The abscissa also displays an intercept range scale which was obtained from reference 7. An objective of this analysis was to determine a practical lower limit for sensor angular accuracy, i.e., find an accuracy such that better accuracy does not improve predictor performance. From Figure 10 it can be seen that such an accuracy is quite stringent (at least below 1/8 millirandian). Since this accuracy is probably lower than practical to achieve, no extended effort was made to find the actual lower limit.

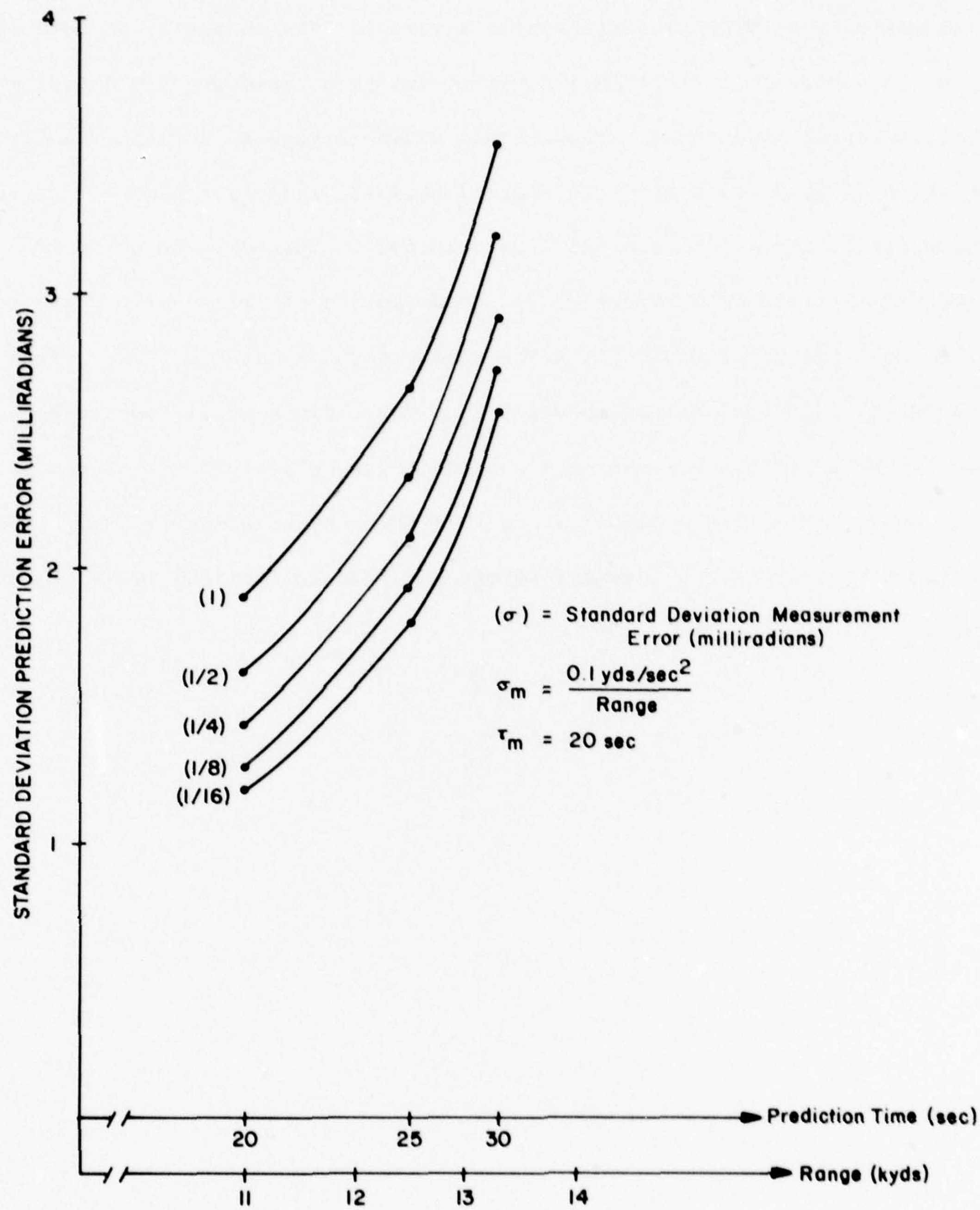

Figure 10. Bearing Direction Prediction Accuracy (surface target)

VIII. CONCLUDING REMARKS

Several techniques have been developed for analyzing the effects of missing range data on tracking filter/predictor performance. These techniques were exercised via the CDC 6700 computer to provide performance estimates for the particular conditions investigated. Computer programs for estimating filter/predictor performance for other conditions of interest are available on request.

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APPENDIX A

RELATING TRANSITION PROBABILITIES TO A PHYSICAL SYSTEM

The purpose of this appendix is to relate the transition probabilities of equation (5) to parameters describing a physical system. The physical system selected to describe the missing data process is shown in Figure A-1. Basically, the selected system consists of a zero mean unit variance 1st order Gauss Markov process driving a threshold detector which determines if a measurement is available at a particular time step. The 1st order Gauss Markov process is conveniently described by its autocorrelation function

$$R_x(n) \triangleq E[x_k x_{k+n}] = \rho^n$$

From the autocorrelation function, ρ can be specified in terms of an autocorrelation time constant, τ, by

$$\rho = \varepsilon^{-\frac{\Delta T}{\tau}}$$

where ΔT is the sample time interval $t_{k+1} - t_k$.

The specific objective of this appendix is to relate the matrix T of equation (5) to the physical system parameters τ (autocorrelation time constant) and T (threshold value).

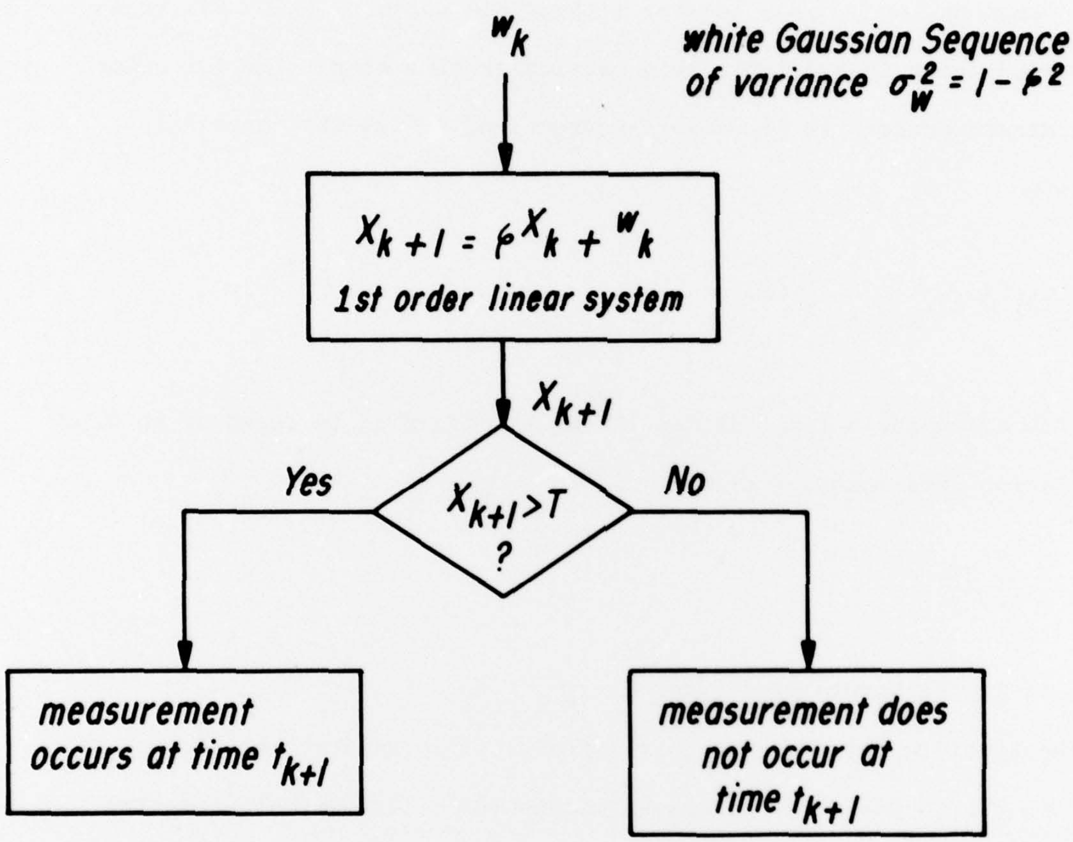

Figure A-1. Physical System with Threshold Detector

Solving for T_{11}

The upper left element of T (equation (5)) is defined by

$$T_{11} \triangleq \text{prob}\{m_{k+1} | m_k\} \equiv \text{prob}\{x_{k+1} > T \mid x_k > T\} \tag{A-1}$$

where prob{a|b} denotes the probability that event "a" occurs given event "b" occurred. The expression (A-1) can be expanded via Bayes' rule to yield

$$T_{11} = \frac{\text{prob}\{x_{k+1} > T,\ x_k > T\}}{\text{prob}\{x_k > T\}} \tag{A-2}$$

where prob{a,b} denotes the probability of event "a" and "b" both occurring. The denominator term of equation (A-2) is simply the unconditional probability of receiving a measurement and is given by

$$\text{prob}\{x_k > T\} = \int_{x_k=T}^{x_k=\infty} N_{x_k}(0,1)\,dx_k$$

where (A-3)

$$N_x(\mu,\sigma^2) \triangleq \frac{1}{\sqrt{2\pi\sigma^2}}\, \varepsilon^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Applying Bayes' rule again, the numerator term of equation (A-2) eventually becomes

$$\text{prob}\{x_{k+1} > T,\ x_k > T\} = \int_{x_k=T}^{x_k=\infty} N_{x_k}(0,1) \int_{x_{k+1}=T}^{x_{k+1}=\infty} N_{x_{k+1}}(\rho x_k, \sigma_\omega^2)\,dx_{k+1}\,dx_k \tag{A-4}$$

where $\sigma_\omega^2 = 1 - \varphi^2$. Combining equations (A-4), (A-3) and (A-2) yeilds the final expression

$$T_{11} = \frac{\int_{x_k=T}^{x_k=\infty} N_{x_k}(0,1) \int_{x_{k+1}=T}^{x_{k+1}=\infty} N_{x_{k+1}}(\varphi x_k, \sigma_\omega^2)\, d_{x_{k+1}}\, d_{x_k}}{\int_{x_k=T}^{x_k=\infty} N_{x_k}(0,1) d_{x_k}}$$

The parameters φ and σ_ω^2 are related to the physical parameters by

$$\sigma_\omega^2 = 1 - \varepsilon^{-\frac{2\Delta T}{\tau}}$$

$$\varphi = \varepsilon^{-\frac{\Delta T}{\tau}}$$

and ΔT is sample interval time step.

T_{22}, T_{12}, T_{21}

It is easily shown that the remaining elements of T are given by

$$T_{21} = 1 - T_{11}$$

$$T_{12} = \frac{(1-T_{11})\ \text{prob}(x>T)}{1-\text{prob}(x>T)}$$

$$T_{22} = 1 - T_{12}$$

$$\text{prob}(x>T) = \int_{x=T}^{x=\infty} N_x(0,1)dx$$

Thus the probability transition matrix T has been expressed in terms of the physical system threshold value (T) and autocorrelation time constant (τ).

APPENDIX B

ACCURACY OF RECURSIVE APPROXIMATE SOLUTION

The purpose of this appendix is to provide an accuracy evaluation of the recursive approximate solution described by equation (16). It is easily shown that the recursive approximate solution (RAS) is exact for the cases of unity or zero probability of receiving measurements. For other cases, however, accuracy evaluation is much more difficult. The evaluation technique described in this appendix consists of comparing the RAS with the exact solution of equation (9) for early transient and steady state responses. The comparisons presented here are based on the following modeling assumptions.

TARGET MODEL

Singer correlated acceleration model (see reference 1) with

$\sigma_m = 0.1$ yds/sec^2 (maneuver level)

$\tau_m = 20$ sec (maneuver correlation time constant)

The measurement sequence is at 16 Hz with standard deviation error of 10 yds.

Experiment #1 (Early Transient Response)

This experiment serves to evaluate the early transient response of the recursive approximate solution. Initial state error covariance is given by

$$P_o = \begin{bmatrix} (10 \text{ yds})^2 & 0 & 0 \\ 0 & (30 \text{ yds/sec})^2 & 0 \\ 0 & 0 & (0.1 \text{ yds/sec}^2)^2 \end{bmatrix}$$

The miss data process autocorrelation time constant (see Appendix A) is 10 seconds. The evaluation presented here consists of a comparison between the recursive approximate solution (equation (16)) and the exact solution equation (9). Expected standard deviation of filter error in position (σ_r) and velocity ($\sigma_{\dot{r}}$) for the two solutions is plotted in Figures B-1 and B-2. Expected standard deviation of acceleration error is virtually identical for both solutions and was therefore not plotted. Figure B-1 results from prob(measurement) = 0.9 while Figure B-2 results from prob(measurement) = 0.5. Note that in both cases the difference between the two solutions is small.

Experiment #2 (steady state response)

Evaluating accuracy of the steady state solution is a bit more complicated. This complication arises because the following principle <u>cannot</u> be easily deduced from the given exact formation (equation (9)) for finite k.

$$E[P_{k+1}] = E[P_k] \Rightarrow E[P_{k+2}] = E[P_{k+1}],$$

where P_k is the filter error covariance matrix at time step k and $E[\cdot]$ denotes expected value.

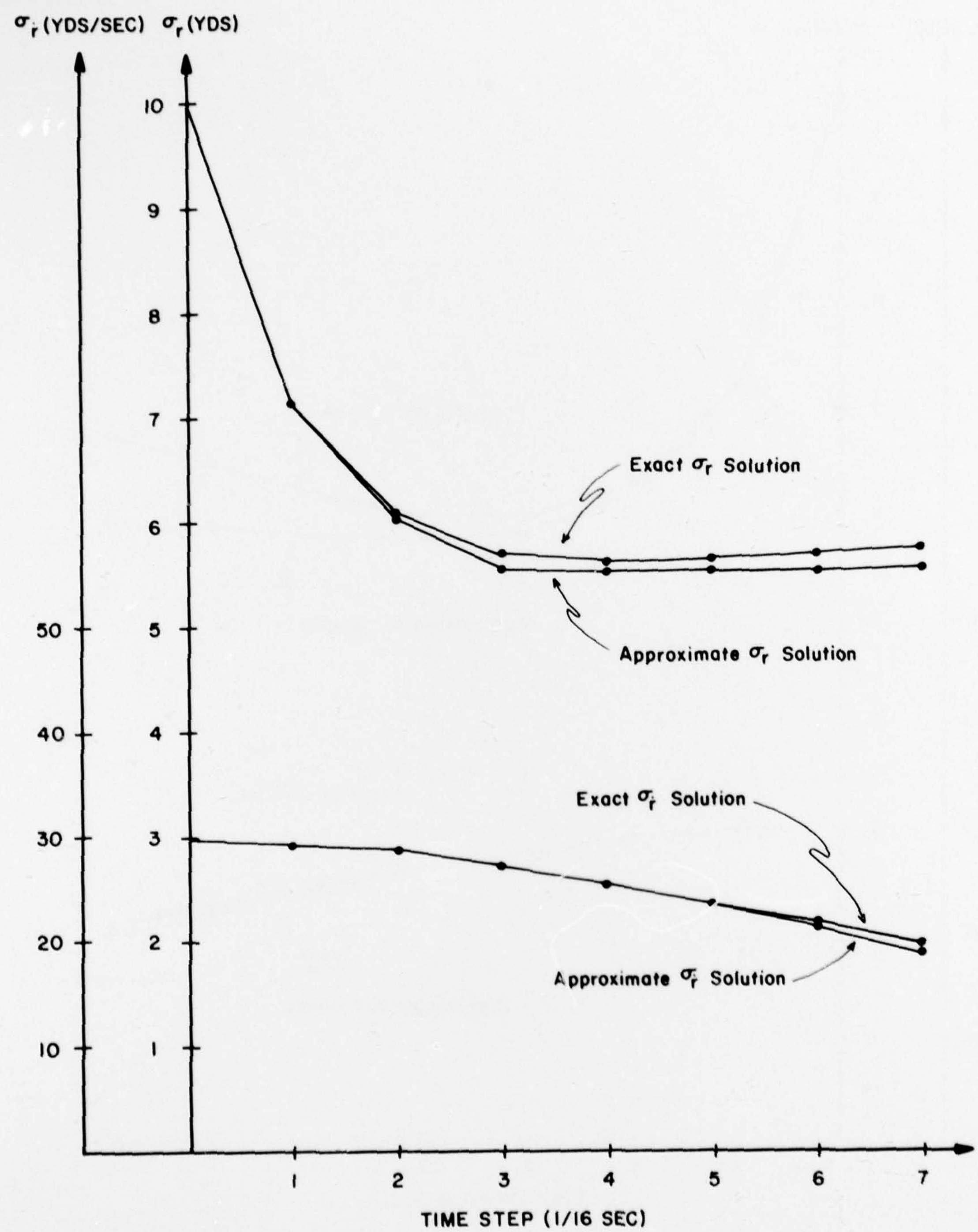

Figure B-1. (Prob (Measure) = 0.9)

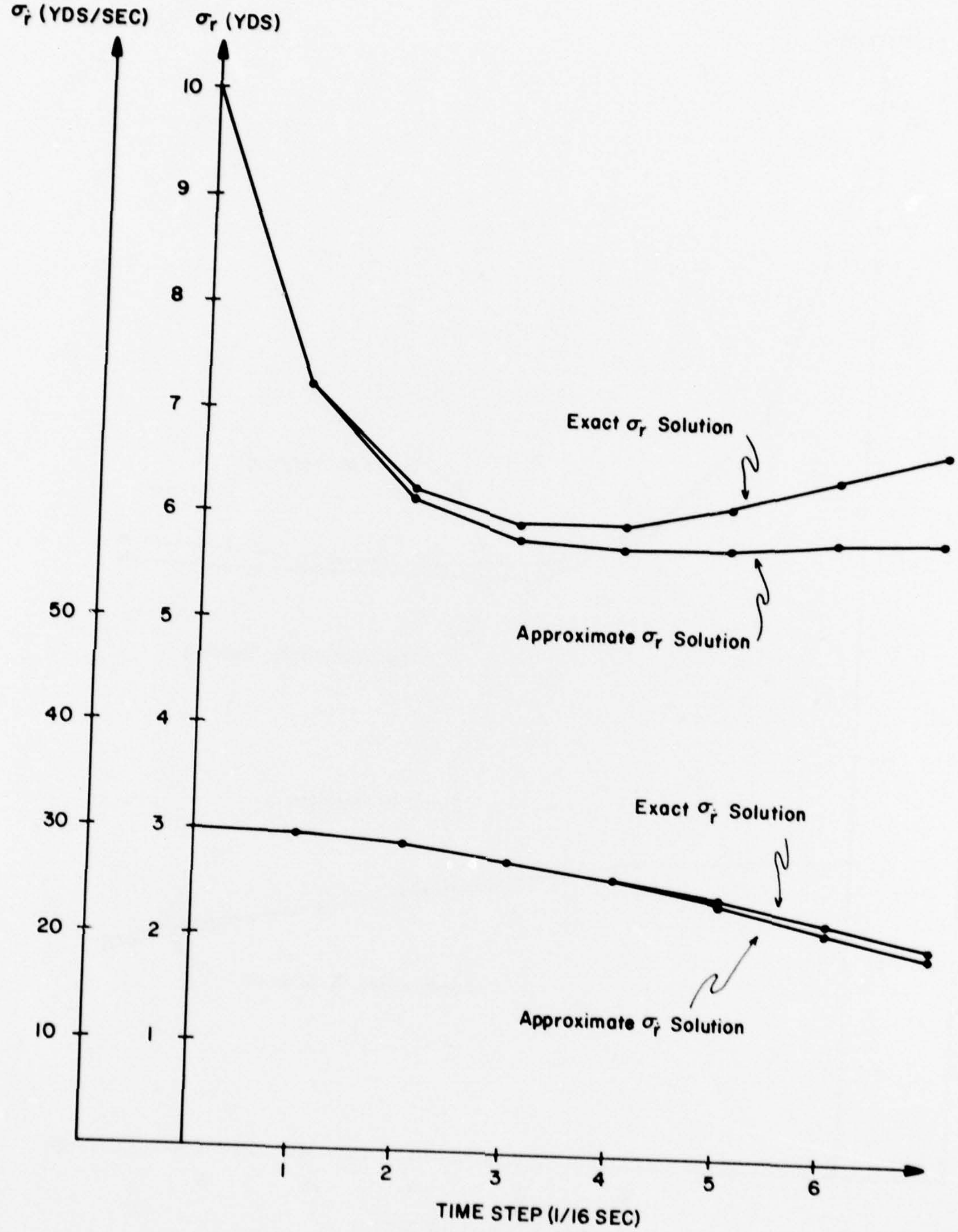

Figure B-2. (Prob (Measure) = 0.5)

Thus recognizing a steady state solution (for finite k) using the formulation of equation (9) is impossible.

The technique used here for testing the steady state solution is as follows.

Step 1. Obtain steady state solution for E[P] using recursive approximate solution.

Step 2. Use the above steady state solution as an initial error covariance matrix for the exact solution (equation (9)) and propagate for several time steps.

Step 3. If after several time steps the exact formulation shows no significant change in expected filter error covariance, the steady state solution of step 1 is judged valid.

The reasoning for the above test is that if the steady state solution of step 1 is significantly in error, a significant change in expected error covariance should show up in step 3.

Using the above procedure and the modeling assumptions discussed earlier yields the results shown in Table B-1.

Table B-1. Modeling Assumptions

Probability of Measurement	RAS Steady State Solution	Steady State Solution Propagated 7 Steps (7/16 sec) using Exact Formulation	Percent Change
0.9	$\sigma_r = 1.68186$	$\sigma_r = 1.68403$	.13
	$\sigma_{\dot{r}} = 0.43213$	$\sigma_{\dot{r}} = 0.43248$	.08
0.5	$\sigma_r = 2.15636$	$\sigma_r = 2.17497$	.80
	$\sigma_{\dot{r}} = 0.49593$	$\sigma_{\dot{r}} = 0.49857$	.53

Notice the discrepancy, indicated by last column of Table B-1, is very small. As in the transient experiment, expected acceleration error variance is virtually identical for both solutions and was therefore not tabulated.

From several experiments, such as the two presented above, it was concluded the recursive approximate solution is quite accurate enough for its intended purposes.